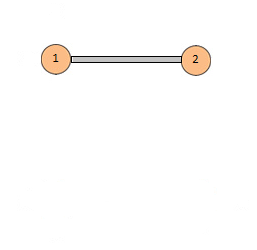
**Question 3**. A barbell for asymmetrically strong people consists of a mass m1 = 10kg connected to mass m2 = 20kg via a 1.5m long, 15kg bar. How far away is the center of mass of the system from the left end of the barbell?

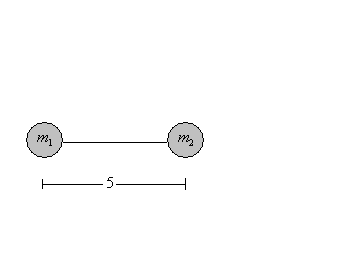


Center of mass coordinate is given by:



**Example**

Consider the following object, with m1 = 2kg, m2 = 2kg.



The center of mass of the object is:



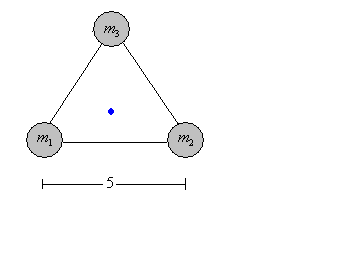
which is the center of the object, as we’d intuitively expect. Suppose now that m1 = 2kg, but m2 = 4kg. We would expect the center of mass to be closer to m2 now, closer to 5.



which is closer as expected.

**Example**

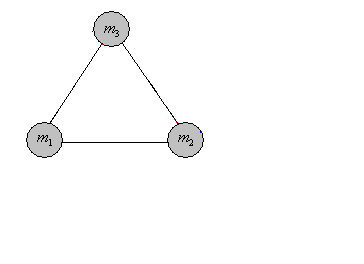
Consider an equilateral triangle. Let m1 = m2 = m3 = m out of simplicity. Where is the center of mass? We’ll take the origin of our coordinates to be m1 again.



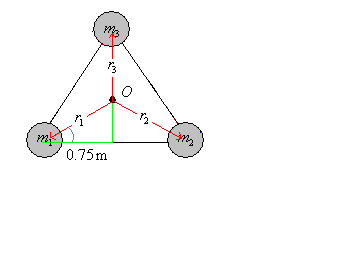
It’s at,



**Question 1.** Consider the following equilateral triangle (side length 1.5m), with m1 = 1kg, m2 = 2kg, and m3 = 3kg. What is the moment of inertia about an axis passing through the center of the triangle, i.e. the point where all the angle bisectors meet?



The center is shown below. The radii r1, r­2, r3 are all equal to each other. To calculate them, we can draw in the following triangle (in green/red).



The base of the triangle is ½ the distance from m1 to m2 (0.75m), and the angle illustrated is ½ of 60˚. So using some trig, r1, the hypotenuse is given by:

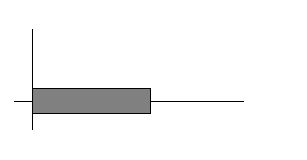


So the moment of inertia is:



**Example**

Consider a uniform board of length ℓ. Where is the center of mass?



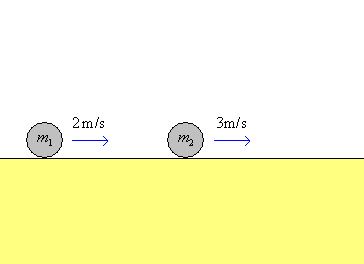
We have,



which is the center of the board again. You will notice the ratio dm/dx introduced in the second line. This is called the linear mass density λ = dm/dx. It is analogous to the density, ρ = M/V of an object, just modified for 1 dimensional cases. For uniform objects λ = m/ℓ. But sometimes the density may not be uniform, in which case it can be a function of x.

**Example**

What is the velocity of the center of mass of the object below. Suppose m1 = 1kg, m2 = 2kg.

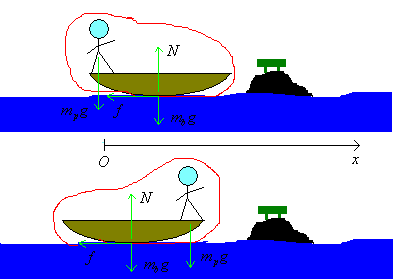


Well,



**Example:**

Suppose you (m = 70kg) are sitting on one end of a 50kg boat 3m long, in a lake. And you see a turtle sitting on a rock about 25cm away from the other end of the boat. If you walk over to the other end of the boat, will you be able to reach it?



Consider you and the boat as the ‘system’ of objects. The external forces acting on you and the boat would be gravity, the normal force exerted by the waters surface, and friction between the water and boat as it slides to the left. The two gravities and the normal force cancel however (b/c you’re not sinking), and the friction is small enough to be relatively negligible. Therefore, there is practically no net external force acting on the system. As such, the center of mass of the system cannot move (because **a**cm = 0). Now, the initial position of the c.o.m. is:



The final position of the center of mass will be:



We also know that

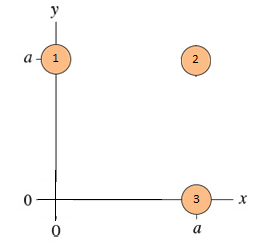


since the person is 1.5m to the right of the c.o.m. of the boat. Plugging this into the other gives,



Thus the person will have moved only 1.25m to the right, and will not be close enough to touch the turtle.

**Question 3**. Three masses lie at corners of a square: m1 = 1kg, m2 = 2kg, m3 = 3kg. Supposing *a* = 5m, what are the coordinates of the center of mass?



According to formula, we have:

